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16 Abstract

This work is an attempt to explain the reasons for discrepancy between the theoretical and experimental values of bubble growth rate in overheated liquid and to provide a brief formulation of the main premises of the growth theory of bubbles in liquid before making a critical analysis of it. In order to simplify the problem, floating up of bubbles is not discussed in this work, moreover its study is based on the results of the theory of behavior of fixed bubbles.

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# THEORY OF BUBBLE GROWTH AND CONDENSATION IN BUBBLE CHAMBERS

L. G. Tkachev

The practical need for development of bubble chamber technology /3* is one of the important reasons stimulating theoretical and experimental research on the behavior of bubbles in liquid, in particular, research on the processes of asymptotic growth and condensation of bubbles.

The main laws governing the behavior of a steam bubble in an overheated liquid have been studied in the works of Plesset and Zwick [1], Forster and Zuber [2]. They demonstrated that bubble growth rate** in liquid depends on a number of factors: inertia and viscosity of the liquid, sound velocity in the liquid and in steam, evaporation velocity of liquid and the rate of supply of heat from the surrounding liquid to the bubble wall. In the area where bubble radius R is much greater than the critical, the factor determining bubble growth velocity is rate of heat transfer into liquid. Because of the low bubble growth rate as compared to sound velocity, the process of heat transmission is discussed on the condition that pressure in liquid equals vapor pressure in the bubble, while liquid temperature on the bubble boundary $T_0$ equals the vapor pressure and is determined from the equilibrium condition of vapor and liquid at a specific pressure.

The results obtained in publication [1, 2] agree well with experimental data of Dergarabedian [3] who investigated bubble growth in overheated water. However measurements of the rate of growth and condensation of bubbles in liquid hydrogen bubble chambers obtained in the works of Alvarez [4] Fabian, et al. [5], Harigel, et al. [6] indicate a definite discrepancy with the results of theoretical studies [1, 2]. The work of Aleksandrov, et al. [7] demonstrated that floating /4

*Numbers in margin indicate pagination in original foreign text.

**One should bear in mind that the stated theory of bubble growth is automatically transferred to the process of their condensation with changes which are only indicated where this is necessary.

up of bubbles significantly influences the velocity of bubble growth and condensation, however its consideration does not eliminate discrepancies with experimental data, and deviation from theoretical quantities is 10 - 30% in the case of bubble growth and reaches 30% in tne case of condensation.

This work is an attempt to explain the reasons for discrepancy between the theoretical and experimental values of bubble growth rate in overheated liquid. It is expedient to provide brief formulation of the main premises of the growth (condensation) theory of bubbles and liquid [1, 2, 8] before making a critical analysis of it. In order to simplify the problem, floating up of bubbles is not discussed in this work, moreover its study [7] is based on the results of the theory of behavior of fixed bubbles. If we ignore floating-up, then we are dealing with a spherically symmetrical problem of searching for the solution to three equations:

$$\frac{4\pi}{3} L\rho' \frac{dR^3}{dt} = 4\pi R^2 k \frac{\partial T}{\partial R}; \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(\rho r^2 v) = 0; \quad (2)$$

$$c_p \rho \frac{dT}{dt} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right), \quad (3)$$

equations of energy preservation with evaporation of liquid from the bubble surface (1), continuity equations (2) and equations of heat conductivity (3) with corresponding boundary and initial conditions

$$T(R, t) = T_0, \quad T(\infty, t) = T_\infty, \quad T(r, 0) = T_\infty \quad (4)$$

where L, $\rho'$ are the evaporation heat and vapor density, k, $\rho$, $c_p$ are heat conductivity, density, heat capacitance of the liquid with constant pressure, r is the radial coordinate of a point in the liquid, t /5

is time, $T(r, t)$ and $T_\infty$ is temperature in the adjoining layer* and at a great distance from the bubble center. Equation (1) is correct on the condition that pressure in the system does not depend on time, and all the quantities included in the equation are computed at the point on the bubble boundary. In solving equations (1) - (3) it is assumed that the thermophysical parameters $k, \rho, c_p$ are constants which do not depend on temperature.** The initial system of equations becomes simpler if we switch to new independent variables and to a new unknown function according to the transform:

$$h = \frac{r^3 - R^3}{3}; \quad \tau = D \int_0^t R^4(t)\,dt, \quad \theta = \frac{T - T_\infty}{\Delta T}. \tag{5}$$

where $D = \frac{k}{\rho c}$ is the so-called temperature-conductance of the liquid, while $\Delta T = T_\infty - T_0^p$ is liquid overheating. As a result of this transform, equations (1) - (3) and additional conditions (4) are transformed to:

$$\frac{dR^3}{d\tau} = \frac{3k\Delta T}{DL\rho} \frac{\partial\theta(0, \tau)}{\partial h}. \tag{6}$$

$$\dot{h} = 0 \tag{7}$$

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial h}\left[\left(1 + \frac{3h}{R^3}\right)^{\frac{1}{3}} \frac{\partial\theta}{\partial h}\right]. \tag{8}$$

$$\theta(0, \tau) = -1, \theta(\infty, \tau) = \theta(h, 0) = 0. \tag{9}$$

/6

In summing the equation (8) difficulties arise since it contains

*The liquid layer at the surface of the bubble in which temperature changes from $T_0$ to $T_\infty$.

**The correctness of this hypothesis for the cases of water and liquid hydrogen is analyzed below.

an unknown function R(r). These difficulties can be avoided if we assume that the ratio $\frac{3h}{R^3} \ll 1$, since in this case one can use the method of the theory of perturbations. It is easy to show that in a zero approximation, solution to system of equations (6) - (8) looks like:

$$\theta = -1 + \frac{2}{\sqrt{\pi}} \int_0^{h/2\sqrt{r}} e^{-x^2} dx \qquad (10)$$

$$R^3 = \frac{6 \Delta T k}{L \rho' D \sqrt{\pi}} \sqrt{r} = 6 A_0 \sqrt{r} \,. \qquad (11)$$

After solution $\theta = \theta(h, r)$ and $R^3 = R^3(r)$ has been found, one can compute the average value of the quantity $\frac{3h}{R^3}$ in order to verify the initial hypothesis regarding its small size. We obtain by definition:

$$3 \left\langle \frac{h}{R^3} \right\rangle = \frac{\int_0^\infty \frac{3h}{R^3} \theta(h, r) dh}{\int_0^\infty \theta(h, r) dh} = \frac{\pi}{4} \frac{L \rho' D}{k \Delta T} \qquad (12)$$

The physical meaning of the quantity $3 \left\langle \frac{h}{R^3} \right\rangle$, as is clear from correlation (12) is the ratio of the volume of the adjoining layer to the bubble volume. Below, Table 1 presents values $3 \left\langle \frac{h}{R^3} \right\rangle$ computed from formula (12) for water and liquid hydrogen in order to compare how applicable the adopted approximation is both in the case of growth and in the case of condensation of bubbles. It also presents dependences on temperature of parameters k, $\rho$ and $c_p$.

The values of the parameters cited in this table were obtained experimentally in publications [3 - 6, 9 - 11]. Because of the promising nature of using neon-hydrogen mixtures to fill bubble chambers [11] data are also cited of a mixture of 90 molar % hydrogen and 10 molar % neon.*

As follows from Table 1, the hypothesis regarding the small quantity $3 \left\langle \frac{h}{\eta} \right\rangle$ is correct for the case of bubble growth, and generally

*The data were taken from publication [10].

TABLE 1. /7

| Наполнитель [a] | $H_2O$ | $H_2$ | 90 % H + 10 % N |
|---|---|---|---|
| $T^\circ_{\text{кип}}$ K рост [b] - 3 | 378 | 25 | 27 |
| $3\langle \frac{h}{R^\beta} \rangle$ рост [b] ($\Delta T = 5$) | 0,05 | 0,15 | 0,01 |
| $3\langle \frac{h}{R^\beta} \rangle$ конденс [c] ($\Delta T = 2$) | 0,1 | 1 | 0,2 |
| $\frac{1}{h}\left(\frac{dh}{dT}\right)$ | $10^{-3}$ | $3.10^{-2}$ | $2.10^{-2}$ |
| $\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)$ | $-2.10^{-4}$ | $-3.10^{-2}$ | $-2.10^{-2}$ |
| $\frac{1}{c_p}\left(\frac{dc_p}{dT}\right)$ | $3.10^{-4}$ | $7.10^{-2}$ | 0,1 |

Key:
a. filler
b. growth
c. condensation

speaking becomes ineffective in the case of condensation. The fact is that the magnitude of overheating $\Delta T$ on which $3 < \frac{h}{R^\beta} >$ depends, according to correlation (12) during bubble condensation is sigrificantly lower than during growth.

The dependence of the quantities $k, \rho, c_p$ on temperature is 10 - 100-fold stronger for liquid hydrogen than for water, so that during overheating by 5° on the adjoining layer, the values $k, \rho, c_p$ change by 15 - 35% in the case of liquid hydrogen and only by 0.1 - 0.5% in the case of water. Consequently, the initial hypothesis about constant thermophysical parameters in the adjoining layer is not justified in the case of liquid hydrogen.

Analysis of the data from Table 1 thus makes it possible to conclude that there are certain grounds for disagreement of theoretical calculations of bubble growth (condensation) rate and the experimental data

for hydrogen. In order to find a more satisfactory agreement between theoretical and experimental data, it is necessary to solve a more general problem: first without assuming that the quantity $\frac{3h}{R^3}$ and equation (8) are small; secondly it is desirable to take into consideration in the initial equations the dependence of heat conductivity, density and heat capacitance of the liquid on temperature.

The dependence of thermophysical parameters k, $\rho$ and $c_p$ on temper- /8
ature severely complicates solution to the initial system of equations. We will therefore ignore the dependence of quantities k and $\rho$ on temperature, since in the case of liquid hydrogen it is weaker than the dependence $c_p(\theta)$. Heat capacitance as a temperature of functions is represented by two terms of the Taylor series:

$$c_p(\theta) = c_{p\infty}(1 + a\theta) \qquad (13)$$

where $c_{p\infty}$ is heat capacitance of the liquid outside the adjoining layer, while the dimensionless parameter of expansion $\alpha$ in the case of liquid hydrogen equals 0.35 with $\Delta T = 5$ and $T = 25°K$.

Thus, we obtain a quasilinear equation instead of equation (8)

$$(1 + a\theta)\frac{\partial\theta}{\partial r} = \frac{\partial}{\partial h}\left[(1 + \frac{3h}{R^3})^{\frac{1}{3}}\frac{\partial\theta}{\partial h}\right]; \qquad (14)$$

and the quantity $\frac{3h}{R^3}$ can adopt random values. Solution to this equation is complicated by the presence of both a nonlinear term and an unknown function R(r). However the properties of symmetry of equation (14) make it possible to simplify the problem. In fact, both equations (14) and equations (6 - 9) are invariant with the transform:

$$r \to m^2 r, \qquad h \to m h. \qquad (15)$$

therefore the solution can be sought for in the form $\theta = \theta(s)$, where $s = \frac{h}{2\sqrt{r}}$. It follows from here that relationship R(r) can be represented as follows:

$$R^3 = 6A\sqrt{\tau} \qquad (16)$$

where taking into consideration the link between variants r and t

$$R = \sqrt{12D}\, A\sqrt{t} \,. \qquad (16) \quad /9$$

where A is a constant not dependent on r, which is determined as a result of solving system of equations (6, 7, 9, 14). Substitution of $\theta = \theta(\eta)$ makes it possible to write these equations in the form

$$A = -\frac{\sqrt{\pi}}{2} A_0 - \frac{d\theta(0)}{d\eta} \,. \qquad (17)$$

$$2(1+a\theta)\frac{d\theta}{d\eta} = -\frac{d}{d\eta}\left[\left(1+\frac{\eta}{A}\right)^{\frac{1}{a}}\frac{d\theta}{d\eta}\right] \,. \qquad (18)$$

$$\theta(0) = -1, \qquad \theta(\infty) = 0. \qquad (19)$$

Equation (18) is a common quasilinear differential equation whose solution depends on two parameters a and A. Parameter a, by definition, must be small as compared to 1, therefore it is simplest of all to search for solution to equation (18) in the form of a series for a. We obtain in a linear approximation:

$$\theta = \theta_1 + a\theta_2 \,. \qquad (20)$$

Since parameter A also depends on a, then in addition to expansion (20) it is necessary to simultaneously have

$$A = A_1 + aA_2 \qquad (21)$$

As a result of expansion, system of equations (17) - (19) is broken down into two systems. The system of equations in a zero approximation for a looks like: /10

$$A_1 = \frac{\sqrt{\pi}}{2} A_0 \frac{d\theta_1(0)}{d\sigma},$$

$$3\sigma \frac{d\theta_1}{d\sigma} + \frac{d}{d\sigma}\left[\left(1 + \frac{\sigma}{A_1}\right)^{\frac{4}{3}} \frac{d\theta_1}{d\sigma}\right] = 0 \qquad (22)$$

$$\theta_1(0) = -1, \quad \theta_1(\infty) = 0.$$

After solution $\theta_1(s)$ and $A_1$ have been found, the system of equations of the first approximation is defined for a

$$A_2 = \frac{\sqrt{\pi}}{2} A_0 \frac{d\theta_2(0)}{d\sigma},$$

$$3\sigma \frac{d\theta_2}{d\sigma} + \frac{d}{d\sigma}\left[\left(1 + \frac{\sigma}{A_2}\right)^{\frac{4}{3}} \frac{d\theta_2}{d\sigma}\right] =$$

$$= -3\sigma\theta_1 \frac{d\theta_1}{d\sigma} + \frac{d}{d\sigma}\left[\left(1 + \frac{\sigma}{A_1}\right)^{\frac{1}{3}} \frac{4\sigma A_2}{3A_1^3} \cdot \frac{d\theta_1}{d\sigma}\right]. \qquad (23)$$

$$\theta_2(0) = \theta_2(\infty) = 0.$$

The obtained equations are solved fairly easily, and solution $\theta_1$ and $\theta_2$ are expressed in quadratures which have a cumbersome appearance and are therefore not cited. After solutions $\theta_1$ and $\theta_2$ have been found /11 the final equations for defining parameters $A_1$ and $A_2$ have the following appearance:

$$-1 + \frac{6A_1^2}{\sqrt{\pi} A_0} \int_0^1 \sigma \exp\left[3A_1^2 \Phi(x)\right] dx = 0. \qquad (24)$$

$$A_0 = \frac{6A_1^3 \int_0^1 \exp[3A_1^2\phi(y)]\,dy \int_y^1 (z^{-3}-1)\,dz\,[-1+\frac{6A_1^3}{\sqrt{\pi}A_0}\int_0^1 \exp[3A_1^2\phi(z)]\,dz}{\int_0^1 \exp[3A_1^2\phi(y)][-\frac{4}{3}y^3+\frac{7}{3}-2A_1^2(2y^{-2}+5y-y^4-9)]\,dy} \quad (25)$$

where $\phi(x) = \frac{-1+3x^3-2x^4}{x^3}$ and $A_0 = \frac{\Delta T \cdot k}{L \rho' D \sqrt{\pi}}$ are the factors determining the bubble growth rate on the conditions that a = 0 and $\frac{3b}{R^3} \ll 1$ . The obtained equations (24) and (25) are solved using numerical integration.

The results of the numerical solution are graphically presented in Figures 1 - 3. It is convenient to select parameter $A_0$ or $A_1$ as the unknown variable. This selection makes it possible to naturally divide the area of change $A_0(A_1)$ into areas corresponding to growth and condensation of bubbles, since the role of the examined effects is immediately visible in both processes.

Figure 1 presents the relationship of the quantity $\frac{A_1}{A_0}$ as a function of $A_0$ which characterizes the influence of the adjoining layer thickness on the theoretical growth (condensation) rate of fixed bubbles. The nature of the relationship allows us to conclude that:
a) consideration for the term $\frac{3h}{R^3}$ in the heat conductance equation results in an increase in the theoretical growth (condensation) rate of the bubble;
b) correction for growth rate is low and lies in limits 8 - 15%, correction for condensation rate of bubbles is significant and reaches 40 - 150%.

The correction emerging with consideration for the dependence of /12
heat capacitance of liquid on temperature is presented in Figure 2 with the help of the quantity $(-\frac{A_2}{A_1})$ as a function of $A_1$. As is apparent from the figure, the correction is negative everywhere, and rises with an increase in the growth (condensation) rate; in the growth area it is

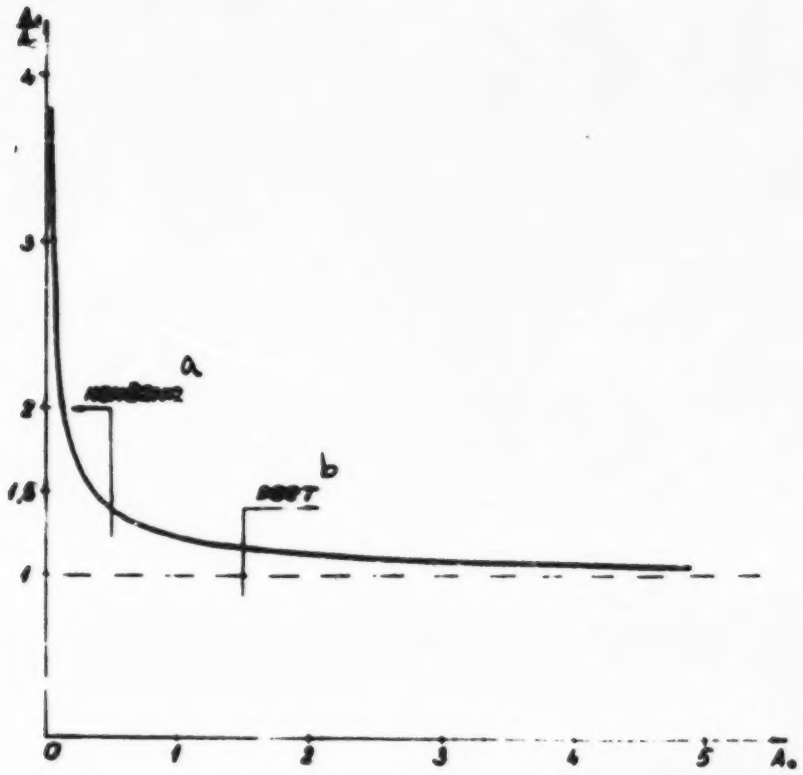


Figure 1. Relative Correction for Theoretical Growth (condensation) Rate of Bubble with Precise Calculation of the Influence of the Thickness of the Adjoining Layer Between the Bubble and the Liquid

Key:
a. condensation b. growth

12 - 15%, in the condensation area less than 10%.

Thus, the total correction for bubble growth rate lies in the area 5 - 10%, the correction for condensation rate could reach 150% and is mainly governed by the high thickness of the adjoining layer.

Because the thickness of the adjoining layer plays a significant role in the discussion of bubble dynamics in liquid, Figure 3 presents the relationship of the quantity $3 < \frac{h}{R^3} > = < \frac{r^3}{R^3} > - 1$ as a

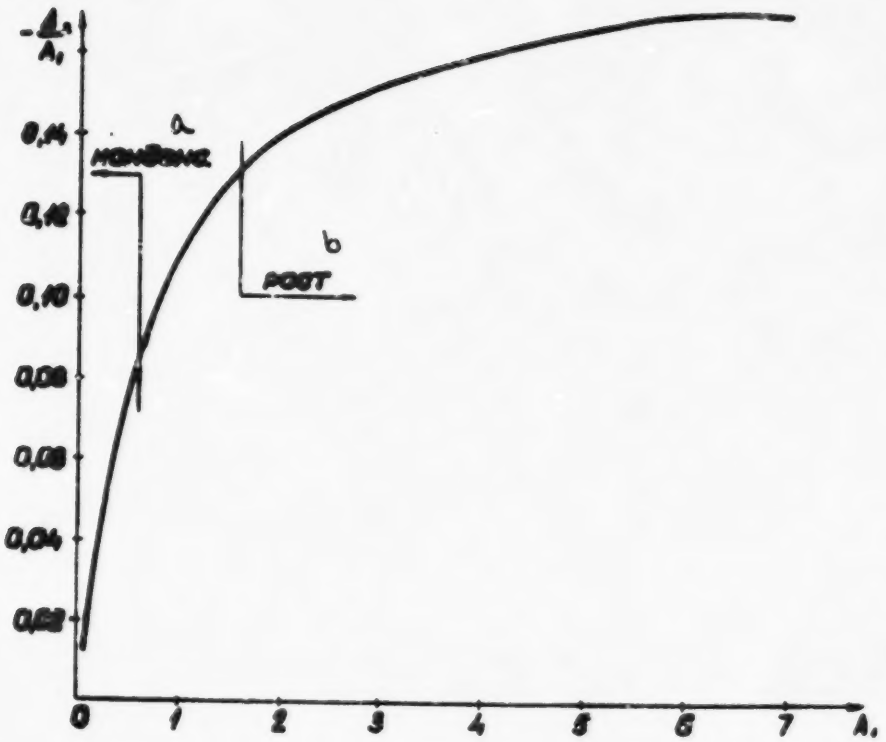


Figure 2. Correction for Theoretical Growth (condensation) Rate of Bubble Governed by the Appearance of Liquid Heat Capacitance on Temperature

Key:
a. condensation b. growth

function of A. Its values were computed according to the definition (left side of correlation (12)), and the dotted curve was obtained using Plesset-Zwick solution (10 - 11) with regard for the relationship between $A_1$ and $A_0$; a solid curve is obtained if solution to equations (22) and correlation (16) is used. It is apparent from Figure 3 that the thickness of the adjoining layer is low during bubble growth processes, and as should be both curves in this area of change in $A_1$ coincide. In the case of condensation, the quantity $3\left\langle \frac{h}{R^2} \right\rangle$ could adopt random, generally speaking, high values, and the Plesset-Zwick solution yields underestimated values of the boundary layer thickness. The initial conclusion drawn on the basis of an approximate discussion of the processes of bubble growth and condensation in bubble chambers thus

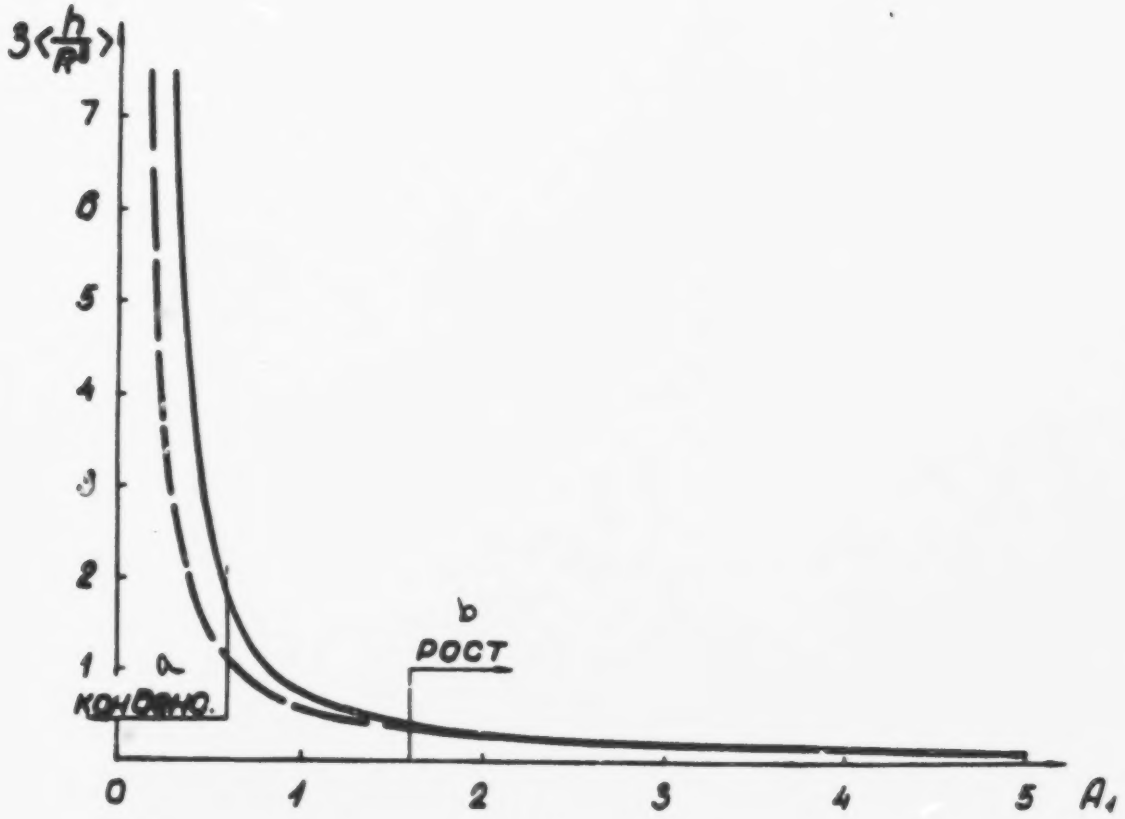


Figure 3. Ratio of the Volume of Adjoining Layer to Volume of Bubble During Growth and Condensation of Bubbles. The dotted line corresponds to the Plesset-Zwick solution, the solid to a solution obtained in this work.
Key:
a. condensation b. growth

is strengthened with a more precise discussion.

In conclusion it is appropriate to compare the results obtained with the available measurements of growth rate and recompression of bubbles in hydrogen bubble chambers.

a) experimental data regarding bubble growth with constant pressure [3 - 6] confirm that $R \sim \sqrt{t}$, as follows from a general appearance of equations (6 - 9, 14).

b) the difference between the theoretical and experimentally computed coefficients of proportionality in this relationship R(t) reaches 30% [6]

cannot be viewed as proof for the incorrectness of the stated approach, /13 since computation of the experimental factors did not take into consideration change in the quantity T, $\rho$ and $c_t$ with adiabatic reduction in pressure from $P_{max}$ to $P_{min}$ [12]. The laws of thermodynamics determine these changes as follows:

$$\delta T = \int_{P_{max}}^{P_{min}} \left(\frac{\partial T}{\partial p}\right)_S dp = \int_{P_{max}}^{P_{min}} \frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p dp \qquad (27)$$

$$\delta p = \int_{P_{max}}^{P_{min}} \left(\frac{\partial p}{\partial p}\right)_S dp = -\int_{P_{max}}^{P_{min}} \rho\left[\left(\frac{\partial v}{\partial p}\right)_T + \frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p^2\right] dp \qquad (28)$$

$$\delta c_p = \int_{P_{max}}^{P_{min}} \left(\frac{\partial c_p}{\partial p}\right)_S dp = \int_{P_{max}}^{P_{min}} -\frac{T}{c_p}\left[\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial c_p}{\partial T}\right)_p - \left(\frac{\partial^2 v}{\partial T^2}\right)_p\right] dp. \qquad (29)$$

where S is entropy of the system, $v = \frac{1}{\rho}$ is specific volume of the liquid. However it is difficult to employ these formulas since it is necessary to know the dependence of density and heat capacitance on pressure and temperature both in the normal and metastable region. Ignoring corrections (27) - (29) does not permit an accurate comparison with the theoretical calculations, since in one case it is impossible to ignore, and in the other to take into consideration effects of the same order.

c) the last remark concerns bubble recompression. It follows from our discussion that in this case the thickness of the adjoining layer $\delta$ = R. However in publication [7] in studying recompression of floating bubbles it was hypothesized without substantiation that $\delta \ll R$. Apparently it is precisely this circumstance which causes the high noncorrespondence between experimental [6] and theoretical [7] data regarding bubble recompression rate and liquid hydrogen bubble chambers.

The important role of bubble floating during recompression is /14 undoubtable; this is confirmed by experimental observations [6]. Consequently it would be important to discuss the dynamics of floating bubbles with regard for the results of this work.

The author considers it his pleasant duty to express great gratitude to L. G. Zastavenko and U. Kundt for stimulating discussions, as well as G. S. Voronov, Yu. P. Merekov, G. I. Selivanov and A. I. Filippov for valuable remarks.

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